# SCATTERING BY LARGE PERIODIC SURFACES: NOVEL NUMERICAL METHOD AND APPLICATIONS

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## ABSTRACT

We present a novel numerical method that enables efficient computation of scattering and emisivity of natural and man-made surfaces arising in the field of remote sensing. We consider periodic surfaces of possibly very large period, to adequately capture the various scales present in them, and we apply our method to study the effect on radar polarization ratios of the presence of a tilling structure in an otherwise random rough surface.

***Index Terms*—** microwave remote sensing, scattering, surface roughness, numerical methods

## 1. INTRODUCTION

The problem of rough surface scattering is of fundamental importance in remote sensing, as new observations and scientific and technological challenges reveal discrepances with existing solutions of scattering problems. In this paper we consider the fact that HH to VV ratios larger than one are observed in the radar backscattering of agricultural soils, on the basis of a new numerical method for the scattering of rough surfaces that contain a tilling structure.

The effect of a tilling structure in the scattering and emmisivity of agricultural soils has been studied by solving Maxwell's equation in a periodic surface formulation [1]. However, the effect of random perturbations in a periodic setting was not considered in that work. On the other hand, the radar backscattering of random perturbations of otherwise coherent structures has been studied in [2, 3], employing analytical approximations whose ranges of validity have not been established yet for this particular type of multi-scale surfaces.

Diverse recurring difficulties prevent more widespread use of numerical methods for the solution of rough surface scattering problems. Such are the cases of resonances arising due to large surface height to wavelength ratios, complex and large geometries wich can include certain periodicity together with random perturbations, and the need to resolve fine scale features of such surfaces. Moreover, high precision is required to accurately account for low-observables – signals which are orders of magnitudes smaller than the incident energy.

To address these issues, we present a new numerical method based on integral equations which achieves super-algebraic convergence at a reduced computational cost. To discretize a possibly random geometry of a decaying auto-correlation, we employ a periodic surface formulation. We consider surfaces of possibly very large period, to adequately capture the various scales present in a random rough surface that also contains a tilling structure.

Our acceleration procedure is based on equivalent sources in parallel faces and FFTs[4], and we extend that procedure to the unbounded case in the present contribution. An intermediate step of our method, that is the computation of the Periodic Green Function in a cartesian grid, is solved by employing the smooth cut-off function introduced in [5] for the treatment of diffraction grating problems. The use of specialized quadrature rules, as described for instance in [6], leads to a high order Nystrm method that has been implemented for the solution of two-dimensional Dirichlet (TE-PEC), Neumann (TM-PEC) and transmission problems (TE and TM) for the Helmholtz equations.

Non-uniqueness of the usual integral equation formulation of periodic problems at certain frequencies known as Wood Anomalies and the ill-conditioning around them could become a severe limitation for numerical methods based on integral equations, specially in the 3D case. This problem can be solved by introducing a Shifted Green Function [7].

We employ our method to study the following qualitative aspect of Radar data of agricultural surfaces, where a considerable number of cases of HH to VV ratios larger than one are observed (**Fig 1**, see [8])

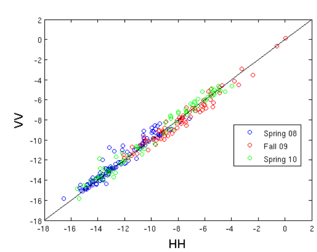


**Fig. 1**. RadarSAT-2 Casselman Campaign

This fact contradicts existing solutions based on gaussian surfaces or alike, whether numerical or approximate methods are used. Empirical methods such as [9] have also reproduced this predicted theoretical behavior. These anomalies could be justified on the basis of subsurface effects such as the layered structure of the soil or particulate scattering within it, or the effect of mild vegetation such as the residue cover. On the other hand, in this paper we show that such anomalies can arise due to surface scattering only, if the geometry of a tilled soil is properly taken into account.

## 2. NOVEL NUMERICAL METHOD

### 2.1. 2D transmission problem

In the case of a translation-invariant surface, Maxwell's equations reduce to Helmholtz equations, and in the case of a dielectric interface the following equations are to be solved

$$\left\{ \begin{array}{rcll} \Delta\psi_1 + k_1^2\psi_1 & = & 0 & \in \Omega_1 \\ \Delta\psi_2 + k_2^2\psi_2 & = & 0 & \in \Omega_2 \\ \psi_1 & = & \psi_2 & \in S \\ \frac{\partial\psi_1}{\partial\hat{n}} & = & \rho\frac{\partial\psi_2}{\partial\hat{n}} & \in S \end{array} \right. \quad (1)$$

With $\rho = 1$ in the TE case (corresponding to H polarization), or $\rho = \frac{k_1}{k_2}$ in TM (vertical polarization).

To solve this system of equations, we employ the Muller system of integral equations that is described in [10], that results from seeking solutions in the form of combinations of single and double layer potentials.

$$\begin{array}{l} \psi_1 = \alpha_1 S_1(\mu) + \omega_1 D_1(\xi) \\ \psi_2 = \alpha_2 S_2(\mu) + \omega_2 D_2(\xi) \end{array} \quad (2)$$

When the boundary conditions are applied to this formulation, the fact that the difference of two hypersingular operators (normal derivatives of the double layer) is a compact operator can be exploited to arrive at a second-kind integral equation, if the constants $\alpha, \omega$ are chosen accordingly.

In order to avoid implementation difficulties associated with normal derivatives of the double layer, an identity due to Maue can be used to replace them by the following

$$\frac{\partial}{\partial s(x)} S\left(\frac{\partial}{\partial s(y)}\phi(y)\right) + k^2\hat{n}(x)\cdot S\left(\phi(y)\hat{n}(y)\right) \quad (3)$$

As this expression only contains tangential derivatives ($s$ denotes the arclength), it can be implemented via numerical differentiation with FFTs[11].

### 2.2. Acceleration of Far Interactions

As a first step, we separate the integration problem into different scales by employing smooth partitions of unity.

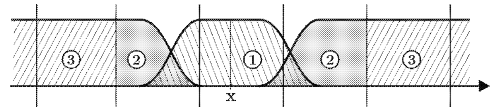


**Fig. 2**. Diagram of smooth scale separation

#### 2.2.1. *Equivalent Sources in Parallel Faces*

In order to convert a surface integral into a convolution, equivalent sources are found by adjusting a number of monopoles and dipoles in parallel faces by seeking to minimize the error of this representation outside a box, by a least squares procedure, following [4]. In a 2D problem we have two sets of parallel faces: vertical and horizontal.

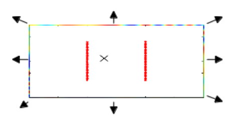

**Fig. 3**. Vertical Equivalent Sources

#### 2.2.2. *Periodic Green Function in Cartesian Grids*


**Fig. 4**. Diagram of Cartesian grids

Now, the interaction of the periodic array of equivalent sources can be taken into account by convolving the equivalent sources with a Periodic Green Function, which has to be evaluated only in two cartesian grids. The convolution can be carried out efficiently by FFTs, but the evaluation of the kernel requires special treatment.

The evaluation of the Periodic Green Function can be performed efficiently by a variety of means, and we chose the following smooth truncation approximation introduced in [5]. To analize the effect of a smooth truncation in the periodic green function, let's consider the simplified case (based on the asymtotic properties of the Hankel function) of computing the infinite integral

$$I_{ex} = \int_0^{+\infty} \frac{e^{ik_n x'}}{\sqrt{x'}} dx' \quad (4)$$

and we consider the following alternatives for its trucation

$$I_{H,A} = \int_0^A \frac{e^{ik_n x'}}{\sqrt{x'}} dx' \quad (5)$$

$$I_{S,A} = \int_0^A \frac{e^{ik_n x'}}{\sqrt{x'}} S(x', cA, A) dx' \quad (6)$$

The usual trucation corresponds to multiplying by a Heaviside function (denoted $H$), and the smooth truncation we employ consists of multiplying by the following function

$$S(x, x_0, x_1) = \begin{cases} 1 & \text{si } |x| \le x_0, \\ \exp\left(\frac{2e^{-1/u}}{u-1}\right) & \text{si } x_0 < |x| < x_1, \\ 0 & \text{si } |x| \ge x_1, \end{cases}$$

$$u = \frac{|x| - x_0}{x_1 - x_0}$$

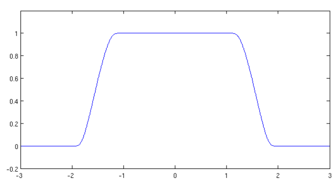

**Fig. 5**. S function with $x_0 = 1$,$x_1 = 2$.

It can be shown via integration by parts, that

$$|I_{ex} - I_{S,A}| \sim O(A^{-p}) \; \forall p$$

that is, super-algebraic convergence, as illustrated in the following numerical example

| A | $\lvert I_{ex} - I_{H,A}\rvert$ | $\lvert I_{ex} - I_{S,A}\rvert$ |
|---|---|---|
| 5 | 7.11315884e-02 | 1.04928986e-03 |
| 20 | 3.55867195e-02 | 3.45741125e-07 |
| 35 | 2.69017479e-02 | 6.86065024e-09 |
| 50 | 2.25077654e-02 | 1.57660945e-10 |
| 65 | 1.97406672e-02 | 7.02890076e-12 |

**Table 1**. Convergence test

#### *2.2.3. Reconstruction of the Surface Fields*

Finally, after performing the previous steps for horizontal and vertical grids (separately), the two fields are put together and the corresponding field in the surface has to be reconstructed. This is a boundary value problem for the Helmholtz equation, which we solve by a least squares fit of plane waves to the boundary values.

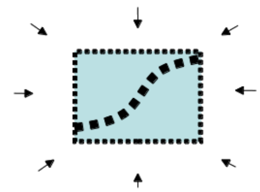

**Fig. 6**. Diagram of plane wave expansion

### 2.3. Convergence test

In order to test the accuracy of our method and the correctness of the present implementation, we conducted convergence tests on dielectric surfaces consisting of random fourier modes, under $40^0$ incidence in C and L band, and verified that errors in the conservation of energy as low as $10^{-10}$ could be achieved.

## 3. A SCATTERING MECHANISM: VV RESONANCE WITH THE TILLING PERIOD

We recall the classical problem of scattering off a sinusoidal geometry of a fixed period and increasing height. As shown below, this gives rise to a strong resonance in VV

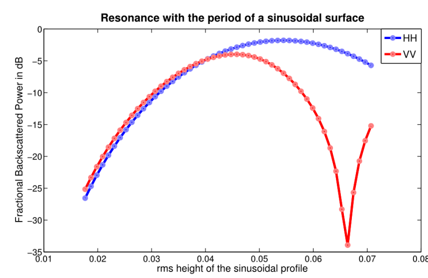


**Fig. 7**. Sinusoidal Surface. Period=80cm, $\lambda$ =25cm

This resonance effect can make the VV return almost zero when the rms height reaches $\frac{\lambda}{4}$. This triggers a polarization anomaly around $\frac{\lambda}{6}$.

### 3.1. Random perturbations of coherent structure

It could be agued that in the case of agricultural surfaces this resonance could fade out completly due to random perturbatios of the geometry. To address such concern we have performed a montecarlo study of random perturbations of a coherent surface

In **Fig. 9** we see that HH to VV anomalies are damped but still persistent under random perturbations. The simulations correspond to C-Band backscattering from a perturbed coherent structure with a period of 15cm, and rms height in the range of 0-4 cm.

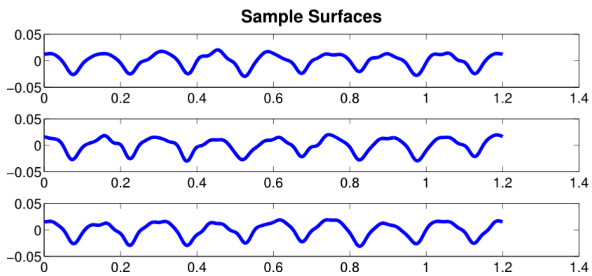


**Fig. 8**. Example surfaces

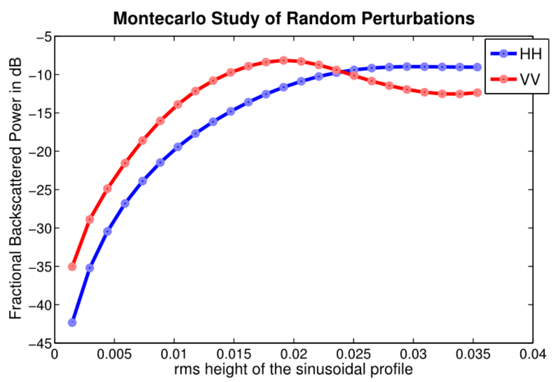


**Fig. 9**. Montecarlo Study of 30 surfaces similar to those shown above

### 3.2. Effect of Dielectric Constant

The preceeding simulations were on the PEC case, which corresponds to a limiting case as the dielectric constant increases. The simulation in **Fig. 10** studies the effect of dielectric constants in the range of $\epsilon = 4-20$ for large sinusoidal profiles in L band, and shows that the same behauviour holds throughout the whole range of dielectric constants found in soil moisture remote sensing with the sole effect of reducing the overall backscattering power, and thus finding that HH to VV ratios larger than one can also ocurr by this mechanism for low values of the backscattered power.

## 4. DISCUSSION AND FURTHER WORK

It is shown that the often overlooked surface-scattering mechanism of the VV-resonance with the tilling period can have an important effect on the polarization ratio. Our simulations show that this effect persists under random perturbations of the surface on both large and small scales and for different dielectric constants, and indicate that polarization anomalies in C-Band could be explained without considering the effect of vegetation or particulate scattering within the soil. For L band the occurrence of resonant agricultural surfaces is rare (to our knowledge, the only exception are the potato fields), but nev-

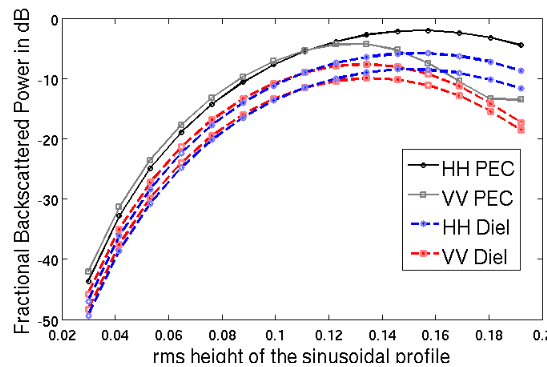


**Fig. 10**. Effect of the dielectric constant

ertheless, the hypothesis of the independence of the polarization ratio on geometrical features of the soil (and therefore it's use as a direct indicator of soil moisture ) is yet to be analyzed with the new methodology. Further analysis in various wavelenghts for different tilling types are in progress.

## 5. ACKNOWLEDGMENT

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